1 more or less

If children are to perform mental addition and subtraction efficiently and with confidence, they need to choose the right strategy, says Mike Askew...

Adele is performing in London.
5003 tickets are on sale.
Tickets sell quickly and 4997 tickets are sold in a week.
How many tickets remain on sale?

On the surface this looks like a routine, not very exciting "word problem" and, to adults, the answer of 6 is quickly obvious. Novice mathematicians – children – find it less clear, because of the distinction between the "action" of the story and the mathematically most effective method of solution. The action of the story is one of taking away – a certain number of tickets are available to start, a number are removed through sales, how many remain? Mirroring this action in the mathematics leads to calculating 5003 – 4997. Setting this out as a standard algorithm then leads to a not very attractive calculation to carry out (although it is the sort beloved of text book writers).

However, in working with the mathematics rather than with the "real" world, it is more efficient to carry out a different calculation. Given the numbers involved it is mathematically more effective to find a difference: 4997 + ? = 5003. This is partly why a spot-the-key-words approach is not most effective; keywords, sometimes, suggest a calculation you could do, but not necessarily the best one.

"MENTAL MATHS IS NOT INTENDED TO BE LIKE MENTAL ARITHMETIC IN THE 1950S, WHERE CHILDREN WERE PRESSURED INTO ANSWERING QUICK-FIRE QUESTIONS AROUND THE CLASS."

Why children may struggle

A teacher and I were interested in whether or not a class of Y4 children would carry out the calculation in line with the real world situation, or whether they would spot that there was a more effective way of calculating.

Working in pairs, many children went straight to setting out a formal algorithm, and were not always successful in getting a right answer. Several fell into the trap of saying "you can't take a smaller number from a larger number" and in carrying out the calculation column by column arrived

Roll ‘em up

HELP CHILDREN DEVELOP THEIR MENTAL MATHS SKILLS

Roll a dice four times and record the scores on the board:

3 2 5 5

Keeping the digits in this order, and inserting addition or subtraction symbols, what numbers can the children make?

For example:

3 – 2 + 5 – 5 = 1   32 + 55 = 87

Variations:

How close can you get to 100? What if you can change the order of the digits?

List a set of target numbers to create:

- **An odd number**
- **A multiple of 3**
- **A prime number**
- **A number between 50 and 60**

As a variation, try rolling a 0 – 9 dice

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at the rather large answer of 1994.

Some pairs chose to use an open number line. They did find a correct solution but their method was based on starting at 5003 and jumping back 4997 to land on 6. Rather tedious and also prone to error. Only one pair of children treated the calculation as a find the difference, figuring out the answer mentally with two jumps of three from 4997.

When this pair was invited to explain their solution to the class, one of the other children quickly and loudly announced “Hang on, that’s not taking away that finding the difference!” We asked the children in their pairs to talk about whether it was okay to figure out the answer in a way that was different from that suggested by the story. In a whole class discussion there was general consensus that what this pair had done was not only acceptable, but highly sensible.

A more efficient way of working

This example illustrates the key difference between working with numbers strategically and working procedurally. The ‘standard algorithms’ are all procedures, that is, the same steps in a calculation are carried out each time regardless of the numbers involved. Therein lies both their strength and weakness. Strength in that you only have to learn one set of steps; their weakness is that those steps may be inefficient for some numbers. Working strategically means starting with a more holistic view – thinking about the numbers involved as well as the operation. Working strategically doesn’t preclude using standard algorithms but it does mean choosing when they are appropriate.

Consider, for example, the difference between calculating 2734 + 3562 and 3998 + 4997. The numbers in the first calculation are not very ‘friendly’ and setting this out as a vertical calculation is a sensible strategy. In the second case, noticing that the numbers are close to multiples of 1000 suggests that the answer is only 5 away from 9000: much more efficient than reaching for paper and pencil.

Choosing the right strategy

A key issue in developing children’s ability in strategic calculation is achieving a balance between building on the informal methods that children can develop for themselves and encouraging them to use particular strategies. Take, for example, the calculation 55 + 37. Children who have developed some confidence in working mentally and a basic understanding of place value, often invent for themselves a method based on splitting both numbers into their tens and ones:

1) 50 add 30 makes 80
2) 5 add 7 makes 12 (possibly done by splitting the 7 into 5 and 2)
3) 80 add 12 makes 92

A slightly more efficient mental method is to split just one of the numbers into tens and ones.

1) 55 add 30 makes 85
2) 85 add 7 is 92 (possibly done by splitting the 7 into 5 and 2) Given that children seem to lean ‘naturally’ towards the first method and the difference in efficiency between the two is marginal, it is tempting to give each method equal value when discussing them with the class. But this could be laying down later difficulties when working with subtraction. For example, what happens with 55 – 37?

The second method above transfers nicely to subtraction. As before split only the second number into tens and ones:

1) 55 subtract 30 makes 25
2) 25 subtract 7 makes 18 (possibly done by splitting the 7 into 5 and 2)

Splitting each number into its tens and ones, and trying to work mentally can be muddling, unless one is confident with negative numbers:

1) 50 subtract 30 makes 20
2) 5 subtract 7 (negative 2)? Can I do that? Maybe I have to subtract 5 from 7 to make 2?
3) do I add the 2 to 20, subtract it, or what?

Build children's confidence

Mental maths is not intended to be like mental arithmetic in the 1950s, where children were pressured into answering quick-fire questions around the class. Developing children’s mental strategies with numbers involves the teacher knowing the range of methods that children might employ and exploring these with the class.

It entails listening to their explanations and encouraging them to discuss their thoughts and mental processes. It means helping them to refine and develop these methods to make them more workable or sometimes adopting someone else’s strategy. And for all of this to happen, the classroom needs to be a safe place in which children can say what they are thinking without fear of being squashed, where they can introduce an idea, however off the wall, and know that it will be carefully considered by others in the room. And the mathematical merits discussed.

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